Evaluation of Michigan Congressional and State Legislative District Plans

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> the analysis of election to statewide office aggregated by district holds more probative value than district-specific races as such. This is because gerrymandering via packing often has the byproduct of discouraging turnout in packed districts. In the extreme, packing a district with one set of partisans and then allowing the district to go uncontested can strongly skew the turnout differential in one partisan direction while skewing the asymmetry effect in the other direction. The countervailing tendencies are largely absent for offices contested statewide (president, governor, attorney general, and the like). Statewide elected offices are not subject to the same within-district turnout incentives as the district-specific races and thus provide a cleaner and clearer reading of a gerrymander effect(McDonald and Best 2015, 318)

A properly constructed baseline model using a combination of prior exogenous elections is, for all practical purposes, identical to more complex regression models, and yield similar results for predictive estimates. In *Whitford v. Gill*, I generated baseline partisanship estimates using presidential election results, demographic data, and geographic fixed effects. The district-level measures produced in this method were almost perfectly correlated with a baseline partisanship measure using prior election data (constructed by an expert retained by the Wisconsin legislature to assist with map drawing), with an r=0.98.[7]

3. Metrics

a. *Partisan Bias*

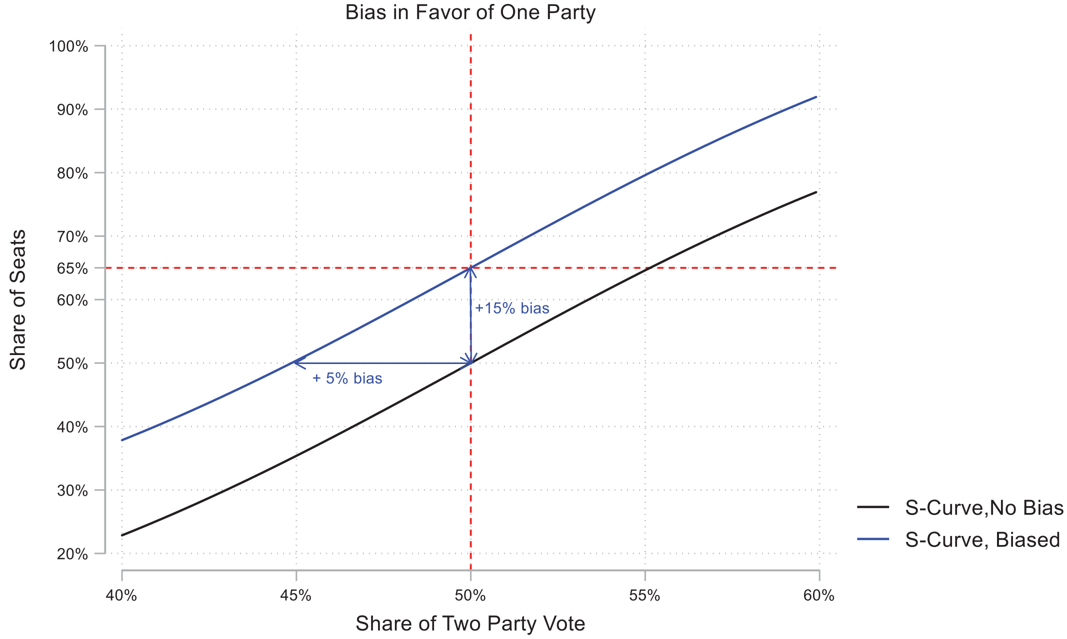
A gerrymandered district scheme is *biased* if it provides a one-sided benefit to a party (Tufte 1983; Grofman 1983; King and Browning 1987; McGhee 2014). In figure 1, all of the

[7] *Whitford v. Gill*, case 15-cv-421-bbc, United States District Court, Western District of Wisconsin.

curves pass through the (0.5,0.5) point, are symmetrical on both sides of 50% of the vote, and none allows a party to secure a majority of seats without obtaining a majority of the vote. In these senses, all are "fair" to both parties. But it is possible that an electoral system results in a general advantage for one side, allowing a party to win a greater share of seats than the other party at most plausible vote shares. Shifting the seats-votes to secure a partisan advantage is "the purpose of gerrymandering" (Tufte 1973, 548).

Figure 2 below shows two seats-votes curves, with the x-axis limited to show vote shares between 40% and 60%. The black line is an unbiased cube-law curve, which passes through the (0.5, 0.5) point, showing that at 50% of the aggregate vote each party wins 50% of the seats; it is the truncated version of the S-shaped curve in figure 1. The blue line shows a biased curve, reflecting an advantage that one party receives in how its votes are converted to seats. At every value of the vote share the party receives, it wins more seats than it wins under the unbiased seats-votes curve. Bias is usually calculated at 50% vote share, because it is the natural dividing point for majoritarian control, and it provides a single point of reference useful in comparing curves that are nonlinear over values on the x axis (Tufte 1973).

Figure 2
Seats-Votes Curves
Bias in Favor of One Party



The two measures of bias – vertical and horizontal – capture different aspects of partisan gerrymandering (McDonald and Best 2015, 315) The vertical measure, captured by subtracting 0.5 from the fraction of seats won at 50% of the vote, is a measure of "seat-denominated bias," (or seat bias), capturing the additional seats a party wins at a hypothetical 50-50 vote. Here, 50% of the vote secures 65% of seats, indicating a positive bias in favor of one party of 15%.

The horizontal measure (indicated by the horizontal blue line) is a measure of "vote-denominated bias" (or vote bias) indicating that the favored party wins 50% of the seats with only 45% of the vote. This shows that the disfavored party would have to win 55% of the aggregate vote in order to win a majority of seats.

Calculating the vote-bias is straightforward: for any set of legislative districts *n*, the number of districts a party must win to obtain a majority, will be $i = (n+1)/2$ rounded up to the nearest integer.[8] If the districts are sorted in ascending order of the vote share of the minority party, the i^{th} district will be the pivotal district.[9] Subtracting the vote share in this district from 0.5, and adding the result to the aggregate vote share that party received, will show the statewide vote share the minority party would need to win in order to win a majority of seats, assuming a uniform swing.

At the state legislative and national congressional level, the party that wins just over 50% of the vote typically wins more than half of the seats, gaining a premium attributable to the nature of single member plurality systems (Tufte 1973). A small bias at 50% does not, by itself, indicate a gerrymander.

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[8] Rounding is necessary in the case of an even number of districts, because the median will be a fraction between the two central district numbers and must be rounded up. In a plan with 14 districts, 8 districts constitute a majority. The median of 14 is 7.5 (the average of the two central quantities, 7 and 8), which must be rounded up to 8.

[9] In Michigan, the pivotal district is the 8th rank-ordered district for a congressional plan, the 56th for the state House, and the 20th for the state Senate (the numbers refer to the rank, not an actual district number).

b. *Partisan Symmetry*

Bias is closely related to *symmetry*, a measure of whether parties are treated equally in their ability to translate votes into seats: if one party wins a particular share of seats at one vote share, the other party will, under partisan symmetry, win the same share of seats if it receives that share of the vote. In Figure 2, this is reflected in the fact that the blue curve is not symmetric around 50% of the vote. The advantaged party wins 65% of seats at 50% of the vote, and 50% of the vote with 44.9% of the vote. By definition, then, the disadvantaged party only wins 35% when *it* wins 50% of the vote, and must win 55.1% of the vote in order to win 50% of the seats.

The existence of partisan bias necessarily implies partisan asymmetry, in that no matter what share of the vote an advantaged party receives, it wins more seats than it would have under an unbiased seats-votes relationship. "Partisan bias," write King and Browning (1987, 1251-2), "introduces asymmetry into the seats-votes relationship, resulting in an unfair partisan differential in the ability to win legislative seats: the advantaged party will be able to receive a larger number of seats for a fixed number of votes than will be the disadvantaged party."[10]

Partisan symmetry is a measure of the underlying properties of a redistricting plan, and is a reliable indicator of gerrymandering. Empirically, it is an attractive metric as it captures a key feature of a fair democratic system – that parties (and, by extension, voters) be treated equally by an electoral scheme. Symmetry "is satisfied when a district plan *does not discriminate between the two parties* with respect to the conversion of votes to seats, and vice versa (Stephanopoulos and McGhee 2015, 843). It is a universally recognized indicator of fairness:

[10] *See also* King (1987,789): "[I]n a two-party system, the absence of bias is the situation where each political party is allocated the same proportion of seats for an equivalent proportion of votes."

> Social scientists have long recognized *partisan symmetry* as the appropriate way to define partisan fairness in the American system of plurality-based elections, and for many years such a view has been virtually a consensus position of the scholarly community. We are aware of no published disagreement or even clear misunderstanding of partisan symmetry as a standard for partisan fairness (Grofman and King 2007, 6).

Calculating partisan symmetry is straightforward: using the results from an election or a measure of baseline partisanship, calculate the aggregate vote share and the seat share for the party holding a majority of seats. Then conduct a uniform swing analysis, shifting the statewide vote by the amount needed to give the other party the equivalent vote share, and applying the shift in each district, determining the winner of each district election at the shifted vote percentage. If both parties have the same share of seats at the equivalent vote share, the electoral system is symmetric. If not, the difference in seat shares obtained at the same vote share is a measure of asymmetry (Niemi and Deegan 1978; Grofman and King 2007).

c. *Efficiency Gap*

The Efficiency Gap, proposed by McGhee (2014) and Stephanopoulos and McGhee (2015) directly measures "wasted votes":

> [A]ll elections in single member districts produce large numbers of wasted votes. Some voters cast their votes for losing candidates (and so are "cracked"). Other voters cast their ballots for winning candidates but in excess of what the candidates needed to prevail (and so are "packed"). A gerrymander is simply a district plan that results in one party wasting many more votes that its adversary. And the efficiency gap indicates the magnitude of the divergence between the parties' respective wasted votes. It aggregates all of a plan's cracking and packing choices into a single number.

(Stephanopoulos and McGhee 2015, 849-50).[11]

"Wasted" in this context means that a vote does not contribute to the election of a winning candidate. To return to a simple example, a candidate receiving 90 votes in a district with 100 voters is elected. But that candidate would also win with 51 votes. In the first instance, the additional 39 votes (calculated as 90 – 51) are "wasted" in that they are not necessary to her victory. Similarly, a candidate who receives 30 votes loses; those votes are wasted in that they play no role in determining the winner. The Efficiency Gap is calculated as the difference in the two parties' total wasted votes, divided by the total number of votes cast (Stephanopoulos and McGhee 2015, 851).

For each district, the wasted votes for each party ($W_{A,i}$ and $W_{B,i}$) are calculated as follows. In district *i,* let $V_{A,i}$ = votes cast for party A and $V_{B,i}$ = votes cast for party B.

$$W_{A,i} = V_{A,i} \qquad if\ V_{A,i} < V_{B,i}\ ,$$
$$= \frac{V_{A,i} - V_{B,i}}{2} \qquad if\ V_{A,i} > V_{B,i}$$

and

$$W_{B,i} = V_{B,i} \qquad if\ V_{B,i} < V_{A,i}\ ,$$
$$= \frac{V_{B,i} - V_{A,i}}{2} \qquad if\ V_{B,i} > V_{A,i}$$

Then the Efficiency Gap is calculated as the difference in the wasted votes of parties A and B, divided by the total votes cast, summed for all *n* districts.

$$EG = \frac{\sum_{i=1}^{n} (W_{A,i} - W_{B,i})}{\sum_{i=1}^{n} (V_{A,i} + V_{B,i})}$$

[11] McGhee initially referred to this quantity as "relative wasted votes" (2014, 68).

In this expression, a positive EG means that party A wastes more votes than party B, and a negative EG means that B is wasting more votes than A.[12] Because the net wasted votes are divided by the total votes cast, the EG is expressed in percentage terms for state legislative plans. Stephanopoulos and McGhee express the EG for U.S. House elections in terms of seats, calculated as the efficiency gap multiplied by the number of seats in a congressional delegation (2015, 854).[13]

d. *Mean-Median Vote*

McDonald and Best (2015) argue that a proper measure of partisan gerrymanders should be focused on how votes are treated differentially, not the specific mapping function that translates vote shares into seat shares. The unfairness of gerrymandering stems from the fact that voters are harmed when the weight of each vote is enhanced or diminished by the way that voters are aggregated into districts (McDonald and Best 2015, 315). The metric of a gerrymander, in their view, is not whether a party wins the share of seats that it deserves, but the simpler measure of whether voters are weighted equally.[14]

They propose a mean-median test that compares a party's mean district vote to its median district vote. A party's median district vote percentage is a crucial pivot point, because it is the precise midpoint at which half the seats lie above that percentage, and half below. By definition, the median vote for the party holding a majority of seats must be greater than 50% (since the party must have at least this share of the vote in half of the districts), while the minority party median vote must be below 50%. A median greater than the mean

[12] The Efficiency Gap can also be calculated using aggregate seat and vote shares, using the simplified formula:

$$EG = (Seat\ Margin - 0.5) - (2\ x\ Vote\ Margin)$$

This converges on the individual seat calculation as turnout across districts becomes equal.
[13] In a state with 15 congressional seats, for example, an Efficiency Gap of -10% would translate into -0.1 x 15 =-1.5 seats.
[14] Best et al. (2018) also argue that because the mean-median test imposes no assumptions about how many seats a party *should* have won compared to a neutral baseline, it is the most direct metric of unequal voting power.

indicates a degree of asymmetry in district lines (arising through packing and cracking): "Where the median is shown to be persistently higher or lower than the mean, a district plan is stacked against one set of partisans" (McDonald and Best 2015, 329). When partisans are packed and cracked – the essence of a partisan gerrymander – the mean vote for the minority party will always be larger than the mean.

Wang (2016) uses the 2012 U.S. house elections in Pennsylvania to demonstrate the phenomenon. Of the state's 18 congressional districts, the Democratic vote share in each, when sorted from smallest to largest (seats won Republicans are Red, Democrats blue) are as follows:

Table 4

	District Rank Order	Democratic Share of Two-Party Vote %
Least Democratic	1	34.4%
	2	36.0%
	3	37.1%
	4	38.3%
	5	40.3%
	6	40.6%
	7	41.5%
	8	42.9%
	9	43.2%
	10	43.4%
	11	45.2%
	12	45.2%
	13	48.3%
	14	60.3%
	15	69.1%
	16	76.9%
	17	84.9%
Most Democratic	18	90.6%

Democrats won just 5 of the 18 seats (28%), even though they received a majority of the statewide vote: the mean of these 18 district vote shares is 51.0%.[15] The median Democratic vote share is 43.3%.[16] The difference – the mean minus the median – is approximately -7.7 percentage points, and indicates asymmetry.

The mean-median difference is also apparent from examining the distribution of vote percentages (Wang 2016, 1300-01).[17] In Pennsylvania, there were 9 seats where the Republican received between 50 and 60 percent of the vote, compared to zero Democratic candidates with those percentages. Four Democrats received more than 65% of the vote, compared to zero Republicans. Taken together, this is an indicator of packing (concentrating Democratic voters in districts where they constitute large majorities) and cracking (spreading other Democratic voters into districts where they constitute minorities).

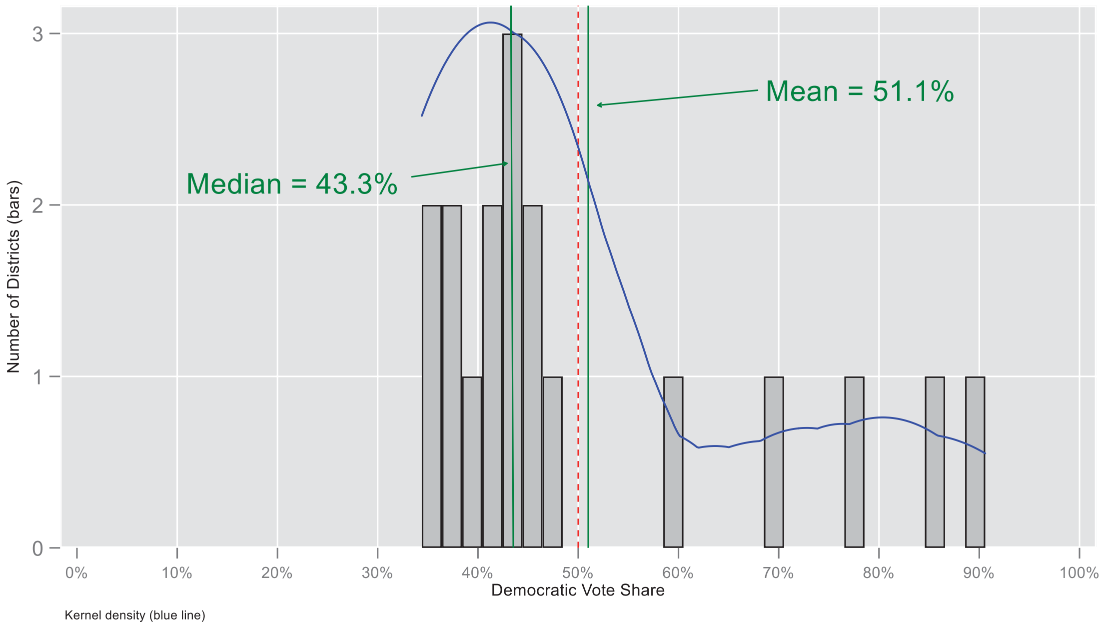
A histogram shows this visually:

[15] The mean-median test weights all districts equally, and does not take differences in turnout into account across districts. The actual statewide 2-party vote split in 2012 House elections was 50.8% -49.2% Democratic.

[16] In a data set with an even number of observations, the median is the mean of the two central values (here, the mean of the 9th and 10th rank-ordered values, 43.2% and 43.4%, or 43.3%).

[17] Gelman and King Stem (1990, 276-77) use stem-and-leaf plots (in effect, histograms and kernel density plots).

Figure 3
Distribution of Pennsylvania U.S. House Districts
2012 Election Results



The skewness – itself a function of packing and cracking –is obvious (note that in this graph, all bars left of the red line at 50% Democratic vote are Republican wins, while the five bars to the right are Democratic wins). Republican-majority districts are concentrated in the 35-50% range (and even more heavily in the 40-49% range), while Democratic-majority districts are all above 60%.[18] The average Democratic winning percentage (the mean for all districts where the Democratic candidate received > 50% of the vote) is 76.4%, while the average Republican winning vote percentage is 58.7%.[19]

Advocates of the mean-median measure argue that it more accurately classifies both gerrymanders and non-gerrymanders (Best et al., 2018).[20]

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[18] In this graph, the blue line is a kernel density, which is a continuous approximation of a discrete histogram.

[19] The efficiency gap in this plan is -23.7 using the individual district data, and -26.2 using the aggregate seat/vote formula, converting into -4.3 and -4.7 House seats, respectively.

[20] The mean-median test does not pick up all examples of gerrymandering. Consider the extreme case where one party draws lines such that it wins 53% of the vote in every district, giving it 100% of the seats with 53% of the statewide vote. Such an imbalance is obviously egregious, but in this case the mean and median votes are the same (53%), indicating no skewness or gerrymandering. The Efficiency Gap, in contrast, would detect it. Three percent of the vote for the winners is wasted

e. *Declination*

Warrington (2018) offers an alternative formula for identifying asymmetry, using the shift in the spatial distribution of vote percentages for the parties on either side of 50% of the vote (which is the cut point where seats change from one party to the other). In a randomly generated (or completely neutral) district plan, there is nothing special about the 50% point and no reason to expect any patterns immediately above or below it. In contrast, "[p]artisan gerrymandering, almost by definition, modifies a natural distribution in a manner that treats the 0.5 threshold as special. Accordingly, one approach to recognizing gerrymanders is to contrast the set of values below 0.5 with the set of values above 0.5" (Warrington 2018, 41).[21]

To see this, consider another visual depiction of the 2012 Pennsylvania US House elections. The *declination* is the difference in the slope of the line connecting districts below 50% to those above 50%. The lines are drawn from the median point in the rank order of each set of districts (those above and those below 50%) to the point midway between the two districts immediately below and immediately above 50%. If the two lines depart from the single line drawn between the central moment of each half of the graph, it is evidence of packing and cracking (Warrington 2018; *see also* Nagle 2015).

(the surplus over 50%), while all 47% of the votes for the losers are wasted. Using the formula EG= (seat share – 0.5) – (2 x vote margin), the Efficiency Gap in this instance is 44%.

[21] In a packing and cracking gerrymander, for example, one party's vote share above 50% will be concentrated at a few high values (packing), and at values close to but below 50% (cracking).

inspection of the distribution of district vote shares shows that this was achieved through the classic techniques of packing and cracking. Without exception in any of the plans, Democratic voters have been packed into districts where they constitute safe majorities, while they have been cracked in others to allow Republicans to win with comfortable but not overwhelming margins. These patterns are observed both prospectively, using data from 2006 to 2010 elections, and empirically, using data from 2012 to 2016. Over a ten year period and 6 electoral cycles, the asymmetry and bias have persisted.

The existence of majority-minority districts does not explain the extent of the observed bias and asymmetry. When these districts are excluded, the asymmetry and bias remain, indicating that even outside of these specific districts district lines create partisan advantage.

The demonstration maps are much more balanced, demonstrating that the partisan nature of the enacted maps was not necessary (the fact that they can be drawn using automated and neutral criteria, by itself, constitutes an "existence proof" of a neutral map and demonstrates that political geography does not explain the partisan character of the enacted maps). Nearly all of the metrics show less asymmetry and bias, and when observed are largely the consequence of majority-minority districts. Some measures are also a function of the high degree of competitiveness in the demonstration maps, with many districts very close to 50% of the vote baseline, such that very small changes in the vote can flip districts between the parties.

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